Historical Perspective on Estimation Techniques for Position and Gravity Survey with Inertial Systems

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Introduction

INVESTIGATIONS into the application of inertial technology to geodetic surveys have covered a span of at least 25 years, involving a number of private companies and government and university laboratories. The principal agency sponsoring this work in the United States has been the U.S. Army Engineer Topographic Laboratories (USAETL), whose original efforts began in the early 1960s.[1] In 1965 the Guidance and Control Systems Division of Litton Industries began its initial tradeoff studies on the Position and Azimuth Determining System (PADS) under USAETL sponsorship.[2-4]

The original estimation problem posed for the PADS was to design software to process discrete measurements from an odometer, or a laser-velocimeter, of the error in system-computed velocity observable at vehicle stops (subsequently called Zero-Velocity Updates or ZUPTS), so that accuracies of 20 m in level position, 10 m in elevation, and 0.3 mrad in azimuth were preserved over a 6-h open-traverse mission. The solution obtained was that of two independent Kalman filters, one for control of the errors associated with the local-level navigation axes and the other for control of errors associated with the vertical axis. Periodic vehicle stops were required, depending on the mode of system operation, to make system corrections with the Kalman filter(s) of the observed velocity error. The design was verified for all three modes of operation via simulation with extensive system error models and successfully field-tested in 1972. The first paper available to the general public on the basic principles for control of system error in the design of the PADS, along with simulation and test results was published in the same year. As in practice the stopping of the vehicle every 10 min to make ZUPT corrections is not excessively inconvenient, the principal operational mode for the PADS has become the unaided, free-inertial mode during vehicle travel. Also contributing to this result was the fact that the laser-velocimeter was an expensive sensor which gave the vehicle a distinctive signature, although it allowed the travel period to be extended to 1 h. Utilization of the odometer has also diminished as it extended the travel period to only 20 min.

The success of the PADS prototype hardware-testing program for the free-inertial mode in 1972[6-8] stimulated interest by a number of groups on the utilization of this inertial equipment for a higher-accuracy position, elevation, and gravity-vector survey capability in helicopters as well as land vehicles. The higher accuracy was easily obtained by relaxing the constraints imposed by the real-time artillery survey mission. The techniques employed were longer pre-mission alignment and calibration time, shorter travel periods between vehicle stops, longer stop periods during which system corrections of the observed error in system-computed velocity were made, and simple adjustment of the real-time position and elevation survey values, using the errors in these quantities available (only) at traverse closure.[33,35,62]

Early Work in Inertial Position Surveying

This latter interest resulted in the delivery beginning in 1974 of a number of systems based on the prototype PADS hardware, but with minor software and hardware modifications. Users included the U.S. Defense Mapping Agency (DMA), Canadian Department of Energy Mines and Resources (CEMR), U.S. Bureau of Land Management (BLM), and Span, Inc. These users began to report the rather accurate test results they were obtaining in 1975.[11-14] The specific inertial surveyors within this collection of systems have been given various names depending on the user, such as the Inertial Positioning System (IPS), Inertial Survey System (ISS), Spanmark,® and Litton Auto Surveyor System® (LASS I).

These user groups were well versed in the use of adjustment techniques for conventional surveys, and shortly papers began to appear on error models and procedures which could be used to adjust the real-time inertial survey position measurements.[32,36,37] Since the Kalman filter design employed for the original PADS provided real-time position measurements of adequate accuracy for these initial applications, little motivation existed for design change when used with the procedures adopted for enhanced accuracy. For cost and schedule-risk reasons, the real-time implementation of higher dimension, more complex error-coupling mechanizations of the Kalman filter did not find financial support.

Early Work in Inertial Gravity Surveying

Also at the time of the 1972 prototype PADS tests, USAETL expressed interest in the development of a more complete geodetic survey system capable of recovering all the measurable geodetic parameters of level position, elevation, intensity of gravity disturbance, and deflection of the vertical.

The design was constrained to be a reasonably low cost modification of the prototype PADS mechanization. A study performed in 1973[9] established a rather simple modification to the operational level axes real-time Kalman filter to achieve this purpose, along with estimates of the potential accuracy

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that the system could obtain for single traverse surveys with post-mission smoothing. This design change deleted the gyro-drift rate states from the Kalman filter so that change in the direction of the gravity vector relative to the mathematical figure of the Earth observed at vehicle stops was contaminated primarily by integrated gyro drift and (necessarily) the accelerometer noise. Correction for these errors in the real-time deflection-change estimates is then made, using the difference between reference astronomic measurements at the initial and closure points of the traverse.[34,38,63] No modification was necessary for the vertical axis real-time Kalman filter to measure the intensity of the gravity disturbance. Reports on the initial testing of the Rapid Geodetic Survey System (RGSS) began in early 1975[10] and improvements in accuracy obtained with various hardware and real-time software modifications have continued to be discussed.[15,16,22,26]

Recent Hardware Developments

During the period that initial deliveries of LASS I equipment were being made and investigations proceeded on the potential for gravity vector survey, the requirements of the U.S. Army dictated that certain changes be made to the prototype hardware to obtain the production model for the PADS. These changes were made, and the resulting systems successfully completed testing in 1979. The Litton Auto Surveyor System II (LASS II) based on the PADS production hardware was created. Photographs depicting this equipment are shown in Figs. 1 and 2. Table 1 presents an overview of the development of survey systems at Litton where Litton Auto Surveyor System I (LASS I) refers to the set of systems based on the original PADS prototype hardware. Users of the newer LASS II equipment include CEMR, Canadian Forces (CF), USAETL/DMA, and ITECH. Some minor variations of this system exist, depending on the interest of the user. For highest accuracy, all three channels (as opposed only to the vertical channel) use the low-noise A1000 accelerometer. The present version of the RGSS uses this equipment.

At the Third International Symposium on Inertial Technology for Surveying and Geodesy at Banff, Canada, in 1985, high-accuracy positioning-test results were reported by different user groups.[28-31] In particular, Rueger[29] has obtained accuracies of a few centimeters in a few kilometers or roughly one part in 100,000 of the relative position change. These accuracies approach theoretical accuracies of one to two parts per million predicted a few years ago in Huddle[38] as obtained from a study of limitations accuracy due to system error instabilities and the gravity disturbance.

Estimation Work Related to the Principal Inertial Surveyors

The First International Symposium on Inertial Technology for Surveying and Geodesy at Ottawa, Canada, in 1977 provided the first major public forum for presentation of various techniques for error estimation in intertial systems for survey. Table 2 provides a summary of estimation techniques associated with the four specific systems which were discussed then and continue to be reported upon.

The first entry in the table is the Aerial Profiling of Terrain System (APTS) being developed by Draper Labs for the U.S. Geological Survey (USGS). This system, described conceptually at the 1977 symposium,[21] discussed a 20-state, real-time Kalman filter to process the difference between laser-tracker range and angular measurements to presurveyed retroreflectors and the inertial system estimates of these variables to control-system error propagation. A later paper[25] on system architecture proposed only simple real-time resets to the inertial system position and velocity so that the navigation error would be bounded in real time, allowing the retroreflectors to be acquired in a timely manner. It had been decided to move the substantial estimation problem to a post-flight processor. In this scheme, a significant amount of real-time recorded

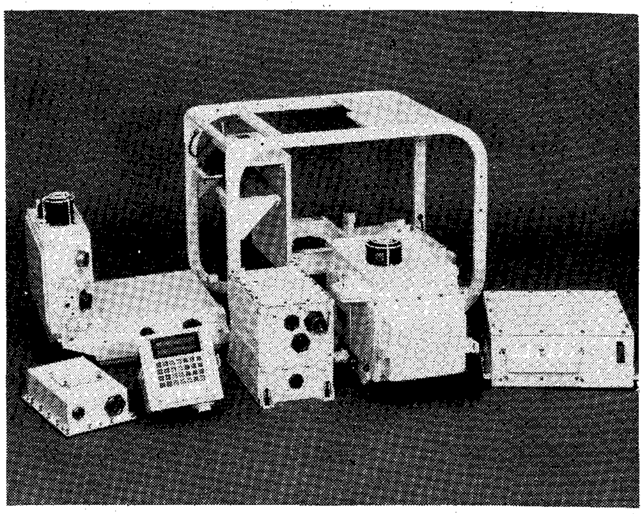

Fig. 1 LASS II hardware elements.

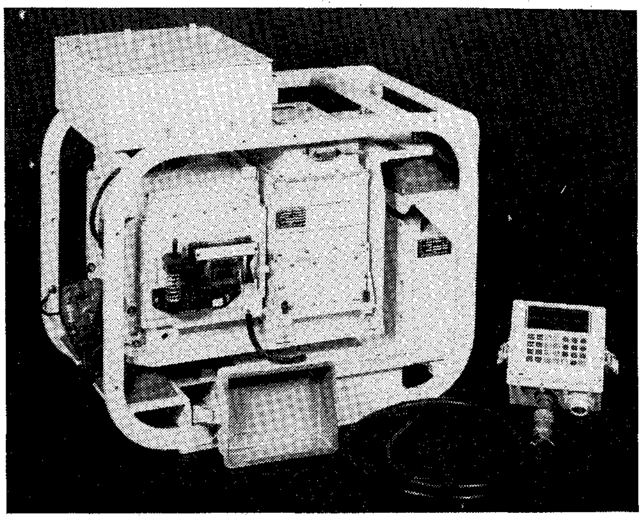

Fig. 2 LASS II hardware assembled in installation fixture.

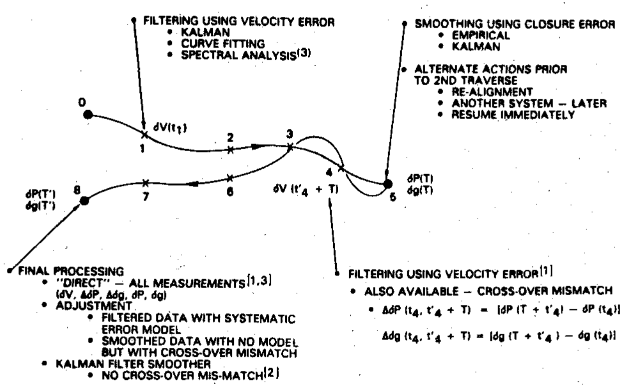

Fig. 3 Summary of various techniques for processing inertial survey measurements.*

*1) Kalman filter presumes $\delta g(P) \rightarrow \delta g(t)$; may not be a problem in the filtering step, as the function of the filter is to provide correction gains to bring the inertial system to an equilibrium state at each stopping point; an unknown in the final adjustment is the effect of correlation of error between spatially near traverses induced by the spatially correlated gravity disturbance.

2) Mismatch of position and gravity estimates at crossover points currently processed by empirical smoothers or multitraverse adjustment programs; a possible technique is Kalman filter with "suspended" states.

3) Discussed in Schwarz[54] and Schwarz et al.[59]

Table 1 Summary of Litton inertial survey system hardware

Designation	User	Units delivered (Sept. 1985)
PADS	U.S. Army	207 + 35 spares (402 to be delivered at 10/month)
IPS-1	DMA[a]	
ISS	CEMR[a]	LASS I: 14 (deliveries since 1974)
	CF[a]	
Litton	BLM	LASS II: 13 (deliveries since 1982)
Auto Surveyor®	ITECH[a]	
Spanmark®	SPAN	
RGSS (3 A-1000s)	USAETL/DMA[a]	

[a]Obtained delivery of LASS II equipment.

Table 2 Evolution of estimation approaches in four inertial survey systems

System	1977 papers	1981 papers	Recent
APTS (Draper)	20 state real-time Kalman filter[21]	Simple, real-time position and velocity resets[25] 29 + 11 per retro. states post-flight processor 10-15 megabytes storage	Post flight processor with 90 or more states[27]
GEO-SPIN (Honeywell)	27 state post-survey batch maximum-likelihood for ZUPTS and position differences at repeat points on traverse[39]	21 state real-time Kalman with smoothing at each control point[24]	
FILS (Ferranti)	Quadratic-polynominal fits to velocity error at stops to remove accumulated position error during travel in real-time[19]	10 state Kalman post-survey filter-smoother[45] Two-stage post-survey program[46] Functional fits to velocity error to correct position, attitude Repeat points on traverse yield unstable parameters Least-square adjustment Experiments with a six-parameter adjustment model[55]	
LASS (Litton)	14-state Kalman real-time level axes error control[17,38] 4 state for vertical axis Individual single-traverse (only) smoothers Position Elevation Gravity disturbance Deflection	Regional adjustment program with collocation for adjustment of filtered data for many traverses[47,52]	Simulation results with RAP improvements[66]

data (10 to 15 megabytes for a 3-h profile) was to be processed by a 29-state and higher dimensional Kalman filter-smoother, depending upon how many retroreflectors (up to 11 states per retro) were to be considered. Recently, this post-flight processor, which is capable of obtaining estimates of 80 or more error states, has been described in more detail.[27]

The last three entries in the table are inertial survey systems that operate in the free-inertial mode between stopping points at which the error in the system-computed velocity vector is observed. The velocity error includes "acceleration error" implicitly, in that a sequence of velocity measurements at a stopping point is usually obtained. Additional measurements typically available to these systems are the reference position and gravity vector at the beginning and end points of surveys traverses. For survey procedures which result in repetition of points (e.g., direct and reverse) or crossing of the traverses, the difference of the computed position and gravity vector obtained at the common encounter points provides further information for a correction process.

The Honeywell Geo-Spin® discussed at the 1977 symposium,[18] proposed the use of a post-mission, batch maximum-likelihood[39] technique to process the velocity errors at vehicle stops, the reference position at traverse end points, and the difference in computed positions at common points of a direct and reverse traverse. The 27 states in the system-error model employed included the customary nine navigation error states plus nine states for both the gyro and accelerometers (three each). Subsequently,[23,24] a 21-state, real-time Kalman filter was proposed for processing velocity errors at stops, with a Kalman smoother to be employed at the closure of each traverse.

The Ferranti FILS[20] discussed at the symposium[19] incorporated quadratic-polynomial fits used to estimate the velocity error between vehicle stops to then estimate the accrued position error in real time. Subsequently, three papers on post-mission processing of real-time survey measurements were presented. One of these papers described a post-mission 10 state (9 navigation error states plus azimuth drift rate) Kalman filter-smoother.[45] The design was tested for three different cases using forward and reverse traverse data for an L-shaped test course near Calgary. The other paper[46] described a post-processing software package that is comprised of two stages. The first stage obtains functional fits of the three-axis velocity error observed at stops and employs this data to obtain corrections to accrued position, platform tilt, and azimuth misalignment. If a forward-and-reverse traverse has been made, differences in position at each common point of the traverse are then employed to obtain estimates of the time variation in azimuth error and the level accelerometer scale factor errors. The second stage is a least-squares adjustment for horizontal position using reference position data to solve for the two fixed-level accelerometer scale factor errors and azimuth misalignment.

An alternative adjustment model for the FILS horizontal position after velocity correction has been investigated.[55] The parameters in this model include two level accelerometer scale factor errors, two azimuth misalignments, and two azimuth drift rates for each level accelerometer.

The Litton LASS I[17,38] has been described briefly above. This system employs two real-time Kalman filters of 14 states and four states for horizontal and vertical axis-error control, with the indicated simple modification for gravity vector survey. Four smoothing programs for correction of error in real-time survey measurements were available at that time which employed only the errors observed at a single traverse closure point. These included:

1) Horizontal position and azimuth misalignment correction, using closure values in horizontal position and azimuth.[62] The adjustment model included the survey point level position error and azimuth misalignment, level accelerometer scale factor errors, initial azimuth misalignment and azimuth drift rate, nonorthogonality of the level accelerometers, and integrated white noise.

2) Elevation correction using the initial and traverse closure value of elevation with the adjustment model including, in addition to the survey point elevation error, the vertical accelerometer misalignments and integrated white noise.[62] A modified version of this smoother, allowing time variation in the vertical accelerometer misalignments, existed for the RGSS mechanization.[65]

3) Deflection of the vertical correction using the reference deflection component change over the traverse.[63] The adjustment model included the survey point deflection errors and the platform gyro-drift rates.

4) Gravity disturbance correction using corrected elevation from the elevation smoother and the magnitude of the gravity disturbance values at the initial and traverse closure points.[64] The adjustment model included trend in the vertical accelerometer bias.

Table 3 Summary of reported work on systematic error models for level position survey

Authors		Error terms								
		Accel. scale factor	Initial azimuth	Initial tilts	Time (t) × distance	$t^{1/2}$	t	$t^{2/3}$	t^2	PE,N
Early models										
Gregerson and Carriere[32]		2, (2)	(2)		2, (2)			2	(2)	
Ball and Voorhees[37]		2	2	2						
Kouba[36]		3	1			2				
Huddle/Litton[33-35,62-65]		2	2			2				2
Later models—via least-squares fit										
Schwarz and Gonthier[49]										
To eliminate systematic error	Forward	2	2							
	Reverse	2	2				2	2		
Hannah[52]		2	2						2	

Table 4 Summary of examinations of systematic error modes using test data

Authors	Remarks	Test Data
Milbert[53]	t^2 coefficients not needed	
(For 0 Gregerson model of Table 3)	Systematic error remains in longitude	SPAN SW Arizona Helicopter 66 and 80 km
Schwarz [41]	A-priori variances for Litton smoother can be selected to obtain near-optimal performance for a specific system.	GSC Ottawa, 100 km L-shaped test course
Schwarz and Gonthier[49]	Examined 6 error models with 4 to 8 parameters by least-squares (L.S.) fit Don't perform position update at forward traverse closure before reverse as it upsets error pattern	GSC Ottawa
Hannah and Pavlis[42]	Examined early error models (G,B,K,L) for 2 control points and concluded performance was similar	USAETL New Mexico, Wash., DC GSC Ottawa
Hannah[52]	Examined 7 error models of 2 to 4 parameters for position, elevation and gravity disturbance and 3 error models of 1 to 3 parameters for deflection by L.S. fit. 15-parameter model for all 6 geodetic variables determined as best	USAETL RGSS New Mexico helicopter and ground vehicle data (Subset of Todd[22] plus more helicopter data)

In 1981 the concept for a more sophisticated post-mission error-correction software, called the Regional Adjustment Program (RAP), was presented with experiments on the application of collocation for improved gravity vector estimation.[47] This program,[57] which incorporates the individual models discussed above into a single program, has been implemented and optimized.[66] In concept, the RAP is capable of processing survey position, elevation, and gravity-vector measurements from a number of traverses throughout a region that have been collected for any number of system realignments and with different inertial survey systems. Further, the differences in the position, elevation, and gravity-vector measurements at all traverse crossover points are processed to obtain excellent self-calibration of the inertial survey systems, as discussed further below.

Later Work in Estimation for Inertial Survey

Beyond the work cited above, there has been a great increase in contributions to the estimation area, beginning with the Second International Symposium on Inertial Technology for Surveying and Geodesy at Banff, Canada, in 1981. A significant amount of work has been carried out by three groups, which include USAETL/DMA, the University of Calgary (UC), and the Ohio State University (OSU). Tables 3-6 identify such work and summarize their contribution in one or more of the following three categories:

1) Appropriateness of parametric models for single-traverse (forward or forward-and-reverse) smoothing or the adjustment of numerous intersecting traverses throughout an area.

2) Performance and properties of various techniques for processing, in addition to the observations for the simple single-traverse category discussed below, the differences of measured positions, elevations, gravity disturbances, and deflections of the vertical at the same physical point, as determined by perhaps different inertial systems after possible new alignments on traverses originating from perhaps different control points. This question therefore deals with techniques for dealing with networks of inertial traverses.

3) Appropriateness of the techniques for processing a subset of the available observations. The types of observations available include the error in system-computed velocity at the vehicle stopping points and the error in system measured position, elevation, gravity disturbance, and deflection of the vertical (as available) at a traverse termination control point. This question, therefore, deals with the simple single-traverse case. This approach is somewhat academic, as in practice the traverse is usually repeated in the reverse direction to ensure reliability of the forward-traverse measurements and to enhance accuracy. Processing of measurement mismatches at the repeated points is sometimes done, obtaining a simplified case of network adjustment as discussed above.

A number of published papers have dealt with more than one of these issues, resulting in relevance to more than one category. This occurred particularly for papers on modeling and network adjustment, as the equality constraint on geodetic parameters at common network points permits higher-dimension systematic-error models.

The first category deals with the initial models being used in the early work of Gregerson and Carriere,[32] Ball,[37] Kouba,[36] Elms and Huddle,[33] Huddle and Lentz,[34] and Huddle[35,62-65] for post-survey smoothing of measurements from a single traverse or from a forward and reverse traverse, as shown in Table 3. Most of the modeling work has been restricted to sets of traverses obtained for a particular test area with a specific system. Conclusions of these various experiments vary and become more divergent as the time of the survey increases. Also, the different test areas and systems used to provide the data may contribute to the diversity of opinion.

Work that has examined these early models and extensions of them is summarized in Table 4. Milbert[53] examined a 12-parameter model ascribed to Gregerson for position smoothing (two level accelerometer scale factor errors and two azimuth misalignments and drift rates, and two coefficients of t^2) and elevation smoothing (two vertical accelerometer misalignments and drifts) and concluded that terms proportional to time-squared (t^2) were not needed and that systematic errors remained in longitude, after smoothing for a forward-and-reverse traverse. The data employed was provided by Span, Inc., for 65- and 80-km test line in southwest Arizona and consisted of 18 forward-and-reverse helicopter traverses.

Schwarz[41] examined the Litton smoothing and found that by changing the a priori variances for the systematic errors, near-optimal performance in position was obtainable. The data employed was provided by the Geodetic Survey of Canada (GSC) for the 100-km L-shaped Ottawa test net and consisted of nine forward and reverse L-shaped traverses and 10 straight traverses. This correponds to an optimal "tuning" of the smoothing software for a specific system. Further, for the same data, Schwarz and Gonthier[49] examined six parametric models by a least-squares fit. They concluded that a simple four-parameter model fit the position error data very well for the forward traverse, but seven or eight parameters were required for the reverse traverse, which was performed after a position update that disrupted the linearity of the error pattern, indicating clearly the presence of initial azimuth misalignment. Empirical smoothing performed with the models indicated the four-parameter model was as good as the others. The weights employed in this model corresponded to level-axes accelerometer scale factor errors of .0027% and .0058%, nonorthogonality of the accelerometer axes of 15 arc-s and initial azimuth misalignments of 38 arc-s. Subsequent empirical analysis of a large amount of data reported recently in Schwarz et al.[59] for the LASS I, yields 1 σ values appropriate for an adjustment model of .0022% and .0010% for the level-accelerometer scale factor errors, 14 arc-s for azimuth misalignment, 5.9 arc-s for the level accelerometer nonorthogonality, and .0007 deg/h for azimuth drift rate.

Hannah and Pavlis[42] examined the Gregerson, Ball, Kouba, and Litton [G,B,K,L] models cited above for position and elevation smoothing for two points on a traverse and concluded that all models performed about the same, but that Kouba's had a slight edge for position and Litton's for elevation. The data employed here was varied, being provided by USAETL from tests in New Mexico and Washington, DC, and from the GSC for Ottawa. For traverses with three control points, a five-parameter (three accelerometer scale factor errors, a common scale factor error for the level accelerometers, and azimuth misalignment) model of Kouba was recommended.

Alternate techniques for processing measurements obtained in a network of intersecting inertial traverses are summarized in Table 5. For a set of RGSS data taken by USAETL in New Mexico, comprising of 11 forward and 11 reverse traverses with alignments at each end and taken with both a ground (~2.18-h traverses) vehicle and a (~.77-h) helicopter (22 total traverses per vehicle), Todd[22] examined the performance for Litton's empirically smoothed position, elevation, gravity disturbance, and deflection of the vertical for different survey procedures. These procedures included single pass over a survey point, averaging double passes using the reverse traverse, averaging two single-crossing passes from different traverses, and averaging two double-pass crossings (from two forward-and-reverse surveys crossing the point). Performance improved approximately by the square-root of the number of passes. The best results were obtained in the helicopter mode, due to the shorter time for error accumulation. The same empirically smoothed data was processed by Zimmerman and Harris[48] by least-squares for the entire network of points, minimizing the differences between the forward-and-reverse traverses while simultaneously minimizing the differences at all traverse crossing points. Also in Todd[22] using this least-squares technique, the RMS errors for the helicopter mode were 10 cm and 21 cm for position, 19 cm for elevation, 0.4 and 0.5 arc-s for the deflection components, and 1.1 mgal for the gravity disturbance. This same multiple-traverse position data, prior to empirical smoothing (Kalman filtered only), was then examined by Todd and Tindall[44] in a least-squares adjustment with 48- and 27-parameter models (four plus and minus accelerometer scale factor errors, 22 initial azimuth misalignments [ground vehicle model only], and 22 or 1 azimuth drift rates). The 48-parameter model performed slightly better than the 27-parameter model, yielding results similar to that for the least-squares adjustment performed on the smoothed data.

Subsequently, Hannah[52] performed an exhaustive analysis of a subset of 12 forward traverses from the USAETL RGSS Kalman filtered ground vehicle data, supplemented with Kalman filtered data from a new set of helicopter traverses comprised of 16 (~.5-h) forward traverses, 8 (~1-h) L-shaped traverses, and 6 (~2-h) L-shaped forward-and-reverse traverses (where the reverse traverse was made immediately without position update). He examined seven different parametric models of two to four parameters per axis for position, elevation, and gravity disturbance and three different parameteric models of one to three parameters for each deflection component. A 15-independent-parameter model in total was determined to be best and was then employed in a least-squares network adjustment for the helicopter traverses. This model employed three independent parameters for each position component (accelerometer scale factor error, azimuth misalignment, and a t^2 coefficient), four parameters for elevation (two vertical accelerometer misalignments and a scale factor error, and a t coefficient), four parameters for the gravity disturbance (three in common with the elevation model but also an independent vertical accelerometer bias trend), four parameters for each deflection component (two in common with the corresponding position error, a level axis drift rate, and an independent t^2 coefficient). Previous work by Hannah[51] with the varied IPS mechanization data (a different Kalman filter mechanization than that for the RGSS data above) of Hannah and Pavlis[42] examined a similarly extensive set of candidate parametric models and arrived at a similar model, but without the vertical-accelerometer scale-factor error (14 parameters) and a parameter proportional to the

Table 5 Summary of reported work on processing of networks of inertial traverses

Paper	Procedure/Remarks	Data
Todd[22]	Compared empirically smoothed: Single pass and average 2 single passes Average forward and reverse traverses Average 2 different forward and reverse traverses Error reduces by $\sqrt{N}$ Also least squares (L.S.) (Zimmerman & Harris[48]) Best Performance for L.S. for helicopter runs Position 10 and 21 cm, RMS Elevation 19 cm Deflections 0.4 and 0.5 s Disturbance 1.1 mgal	11 forward and 11 reverse traverses with both helicopter and ground vehicle as obtained by USAETL in New Mexico
Todd & Tindall[44]	Least-squares adjustment of filtered data 27-parameter model with 22 azimuth errors 4 + and-accelerometer scale factors 1 azimuth drift rate 48-parameter model Above 22 azimuth drift rates Performance of 48 slightly better than 27-parameter model but similar to above least squares. On empirically smoothed data empirically smoothed data	Above USAETL ground vehicle test data
Hannah & Mueller[43]	Least-squares adjustment with 5-parameter model Discovered cross-over points of traverse, obtained excellent calibration of inertial system revealing error in underlying control	GSC southern Manitoba 24 intersecting forward and reverse traverses
Hannah[52]	Least-squares adjustment with 15-parameter model Performance —Position 10 cm —Disturbance 0.9 mgal —Deflection 0.4 arc-s Again discovered potential error in control-elevation	USAETL RGSS 24 roughly straight interlocking helicopter traverses
Arden and Schwarz[56]	Examined procedures for efficient network surveys Concluded strong relative accuracy of interlocking traverses can uncover error in peripheral control Further, peripheral control should not be held fixed—assign proper covariances	None
Schwarz, Arden and English[59,60]	Tested four procedures for network surveys Adjustment of Kalman-filtered measurements obtained superior performance	GSC—expanded Manitoba LASS I Data Set

longitudinal $(\Delta\lambda)^2$ difference-squared (accounting for the long-term effects of azimuth drift) in the latitude model instead of a parameter for t^2.

Additionally, in Hannah and Mueller,[43] position (with a simple five-parameter model), and in Hannah,[51] position and elevation [with the nine-parameter model of Ref. 42 but without the $(\Delta\lambda)^2$ parameter], test results for a network of intersecting inertial traverses were reported. The data was provided by the GSC and comprised of 24 forward-and-reverse approximately 100-km (1.5-h) production traverses in southern Manitoba. In this case, systematic error trends of the internal points of the mesh after adjustment were related to potential systematic error trends in the underlying control. The premise was that the crossover points of the traverses were in themselves sufficient to obtain excellent calibration of the inertial data, thereby revealing deficiencies in the underlying control.

Subsequently, with 24 interlocking helicopter traverses (12 roughly straight, forward, and reverse traverses) from the USAETL RGSS data set, Hannah[52] performed another network adjustment experiment with the 15-parameter model. A potential systematic error was discovered in elevation, making control again suspect. Otherwise, performance of the network adjustment was excellent, approaching the residual error level of the least-squares procedure for model identification, indicating the effectiveness of the network approach. The RMS error performance approached that of the control, being less than 10 cm for position, .9 mgal for gravity disturbance, and 0.4 s for the deflection of the vertical.

Arden and Schwarz[56] have examined alternative network field procedures relative to the standard used by the GSC with the LASS. This procedure includes double forward-and-reverse traverses between four corner control stations of an 80×100 km grid. Northerly directed forward-and-reverse traverses were repeated every 10 to 20 km in the east direction, and there was a single easterly-directed forward-and-reverse traverse at the mid-point of the grid. Consequently, the peripheral and east-directed mid-grid lines of points are encountered four times, while all other points are encountered twice. They found that no advantage was obtained by either executing a criss-cross network of single traverses (encountering all points interior to the periphery twice) or adding a control station at the center of the grid. They did find that the double forward-and-reverse traverses on the periphery could be reduced to single forward and reverse traverses without significant performance loss, hence reducing costs. Finally, they concluded, similarly to Hannah above, that a network of interlocking inertial traverses is so strong in relative accuracy that it will uncover relative error in the peripheral control. Hence, control should not be held fixed in the adjustment but should be assigned proper a priori variances to allow the entire network to move in the adjustment.

Reflecting upon the above body of modeling work and some of that to be discussed below under empirical smoothing, the differences obtained by various contributors seem to lie in the method of dealing with azimuth drift rate for traverses which extend over significant time periods (>1.5 h). The premise in the original smoothing work[62] was that survey position errors ($\delta D_{E,N}$) could be approximately related to a constant azimuth drift rate by the following equations:

$$\delta D(t)_E = \int_o^t d\mu \int_o^\mu a_N(\nu)\nu b_z d\nu = b_z P_E(t)$$

$$\delta D(t)_N = -\int_o^t d\mu \int_o^\mu a_E(\nu)\nu b_z d\nu = -b_z P_N(t)$$

where $a_{E,N}(\nu)$ are the east and north components of vehicle acceleration; $P_{E,N}(t)$ are the dynamic coefficients relating azimuth drift rate to the accelerations that can be generated in real-time in the digital computer and stored for the post-survey adjustment; and b_z is the assumed constant azimuth drift rate over the period of interest. Clearly, the P coefficients can be represented approximately in different ways.

Table 6 summarizes investigations of the appropriateness of methods of processing the available observations for a single traverse. Kouba[36] originally investigated the problem of least-squares adjustment of Kalman-filtered inertial position measurements, using the four-parameter systematic error model cited above. The integrated white-noise model[35] for the accumulation of residual position error induced by (only) system noise and stochastic gravity disturbances after zero-velocity updates, was employed to construct the weight covariance matrix for these errors. Kouba indicated variations required for the multiple-traverse case. Schwarz[40] extended this analysis, deriving the error covariance matrices between different survey points on the traverse for filtered survey measurements and discussed the filter-smoother which can be optimal for the case that position-dependent error sources (e.g., gravity disturbances) can be represented properly as

Table 6 Summary of investigations into various techniques for adustment and smoothing

Authors	Subject
Kouba[36]	Derived a "total" covariance matrix between errors at different survey points, using systematic error plus integrated white noise as the model for filtered survey measurements to be used in a least-squares adjustment: $$\delta P(t) = \Phi(t)X + \int_0^t \xi(\mu)d\mu$$
Schwarz[41]	Derived the total error covariance for (Kalman) filtered survey measurements Presumes gravity disturbance representable as time-dependent process
Schwarz and Gonthier[50]	Discusses the direct (batch) and two-step methods of measurement processing (velocity error, cross-over mismatch and closure errors) Notes lack of formulas for total covariance for optimal smoothed traverses
Gonthier[61]	Showed from simulation analysis that a suboptimal Kalman filter in conjunction with an empirical smoother performs as well as an optimal Kalman filter-smoother for straight, L-shaped, and Z-shaped forward traverses without the added storage and computation costs
Vassiliou[58]	Examined spectral decomposition approach for velocity error processing (suggested by Schwarz[54] in conjunction with empirical smoothing with good performance)

time-dependent functions. This can be achieved by avoiding commonality or proximity of points along the traverses. Schwarz and Gonthier[50] subsequently pointed out a difficulty in obtaining the total error covariance matrix between all the survey points on a traverse that have been obtained by optimal filtering and smoothing. This error covariance matrix is required to integrate properly the survey results with measurements obtained by other survey methods.

Gonthier[61] examined by simulation the interesting question of performance degradation in position error for:

1) A survey system mechanized with a "simplified" 12-state Kalman filter to control the effects of stochastic noise sources with ZUPTS, in combination with a six-parameter empirical smoother (two level-axis accelerometer scale factor errors and two misalignments, and an azimuth misalignment and drift rate) to eliminate these systematic errors, which are not effectively observed by the Kalman filter with the zero-velocity measurements. This is similar in concept to the LASS mechanization.

2) An "optimal" 16-state Kalman filter-smoother (the seven navigation errors, three [variable] gyro drifts two [variable] acceleration errors and the four systematic errors of accelerometer scale factor error and mechanical misalignment, as cited above).

Simulations were performed for straight, L-shaped, and Z-shaped traverses. The conclusion was that the two mechanizations produced almost identical performance over the traverses. The simplified filter and empirical smoother is attractive from a practical point of view, as it obtains this performance without the extensive storage of data and computation required to implement the optimal smoother.

Vassiliou[58] has examined off-line for three forward-and-reverse traverses (~ 2 h) of FILS data for the 45-km, L-shaped test course near Calgary, the position and elevation performance of the method of spectral decomposition suggested by Schwarz[54] for the processing of the zero-velocity measurements in combination with a 10-parameter model (the three accelerometer scale factor errors, the six possible misalignments of their sensing axes, and a common drift parameter for all three axes). Spectral decomposition first obtains a least-squares fit of the error in system-computed velocity to a set of theoretically derived basis functions involving sinusoids of the Schuler frequency and Earth rate. Residual systematic error in the velocity samples are then represented as a summation of low-frequency harmonic sinusoids, as determined from a discrete Fourier transform applied to the residual data. The unfiltered position data is then corrected with the integrals of the obtained velocity error functions. In these tests, position error (prior to smoothing) ranged from 5 to 25 m. Error after correction based on the mismatch of position measurements between the forward-and-reverse traverse at intermediate points, and after empirical smoothing based on the position misclosures at the end of the forward-and-reverse traverses, were 15, 30, and 10 cm, respectively, for latitude, longitude, and height. These errors were less than the errors obtained with the suboptimal 10-state Kalman filter-smoother[45] which processed (necessarily) a subset of the same data.

Clearly from the above discussion, a proliferation of techniques has resulted for processing the measurements available from an inertial surveyor. For the observed error in system computed velocity at vehicle stops, the techniques include 1) Kalman filtering, 2) curve filtering, and 3) spectral decomposition. For the observed single traverse mis-closure in the composition and gravity vector at control points, the techniques include 1) Kalman smoothing, 2) empirical smoothing, or 3) no smoothing if further processing of multiple traverses is to be performed. Further, for the observed differences in the position and gravity vector at common points of multiple traverses, the techniques include 1) empirical smoothing or again 2) no smoothing. Finally, for multiple traverses these pre-processed data are subject to subsequent adjustment with a least-squares procedure which either incorporates a systematic error model if only filtered data are employed, or else a least-squares procedure which incorproates no systematic error model if the data have been pre-smoothed. In either of these latter cases, the differences of the measurements at common traverse points and the mis-closures at control points are minimized.

Schwarz[54] has provided a framework from which to consider this diverse combination of methods relative to nearly-rigorous reference method of simulatneous processing of all the available velocity error, traverse cross-over point mismatches, and closure errors. This reference method is referred to as the "direct" approach and rests on the assumption that the gravity-disturbance vector, which is a position-dependent function, can be transformed to a time-dependent function with appropriate stochastic description. This becomes difficult for forward-and-reverse traverses and networks of intersecting traverses. The practical significance of the correlation between survey errors due to the gravity disturbance vector on "spatially near" traverse points not reflected through such a time function representation remains to be ascertained. This could be significant, as the gravity disturbance vector is a major source of survey measurement error, especially for high-accuracy survey systems. In Schwarz et al.,[59] further detail on this framework has been developed along with the results of experiments performed for a LASS I horizontal position and elevation data set provided by the GSC for survey traverses in southern Manitoba. The study reports on the relative merits of:

1) Adjustment of filtered measurements.

2) Free-adjustment of filtered measurements, with subsequent transformation to control.

3) Adjustment of empirically smoothed measurements.

4) Averaging of empirically smoothed measurements.

They tentatively concluded that the first and third techniques are superior and similar in performance, but that final resolution had to await improvements in the available control. Subsequently,[60] better control-point values to a relative accuracy of 5 cm (1 σ) were obtained via the use of the interferometric GPS technique. With the availability of this information, the first technique of simultaneous adjustment of all filtered measurements was clearly superior for latitude and longitude. The poorer performance of the second method remains problematic and will be investigated further by those authors.

Illustration of Estimation Methods

To illustrate the various estimation techniques discussed above for processing inertial survey measurements, Fig. 3 has been constructed. Beginning at the left side of the figure, the inertial system is aligned and initialized with reference data. The surveyor is then moved along the forward traverse, stopping periodically to observe the error in system-computed velocity, which can be used by various filtering techniques to reduce error propagation. At the closure of the forward traverse at the right side of the figure, additional reference data becomes available. The error in the surveyor estimated position, elevation, and gravity-disturbance vector observed at closure can be used by an empirical or Kalman smoother to correct the survey measurements made at intermediate points along the forward traverse. Azimuth data may also be available but is seldom used in operational surveys today.

At the traverse closure point, the inertial system may be realigned before proceeding with the further collection of survey measurements, or proceed immediately with the survey. An alternate possibility is that another surveyor may be used to collect measurements from the traverse termination point at some different time.

On the second traverse, velocity error is observed at vehicle stopping points and used to control error as in the forward traverse. Further, however, as common points are encountered on the reverse portion of the traverse, mismatches between the position, elevation, and gravity-disturbance measurements obtained on the forward traverse provide additional measurements for error control. These measurements are used by some empirical smoothers to solve for error instabilities when the same inertial system is being used, and in all adjustment methods as the adjustment model dictates.

Note that the processing of these differences is not accommodated by a standard Kalman filter-smoother, as covariance information between errors at different points in time is not available. This could be overcome by adding error states to the filter for the position, elevation, and gravity-disturbance measurements at the survey points on the forward traverse,[9] referred to as "suspended" states below, and then processing the mismatches on the reverse portion of the traverse. This presumes that the same system is used on the reverse portion of the traverse and that the spatially correlated gravity-disturbance vector can be represented sufficiently well for our purposes as a time-correlated stochastic process.

At the lower left side of the figure, a final traverse closure point is encountered which completes a very simple network of inertial traverses in our illustration. Here the same reference data are potentially available as at the closure point of the forward traverse. The various procedures noted above can now be employed to obtain final values for the positions and gravity disturbances at the vehicle stopping points.

Conclusions from Literature Review

In reviewing the contributions cited as references for this paper, the following general conclusions are drawn:

1) Suboptimal filtering of various types, whose objective is to control the time-variant sources of system noise using the error in system-computed velocity observed at vehicle stops, can achieve essentially optimal performance.

2) Empirical smoothing, whose objective is to control the effect of systematic error in the survey measurements which are only weakly observable by the velocity error filtering above, can be based upon fairly simple parametric models and obtain essentially optimal performance. Such a model includes accelerometer scale factor errors and misalignments and the instabilitites in these misalignments. Some variation in opinion appears as to how to represent the instability in the accelerometer alignment, which may be aggravated by unknown effects of the gravity disturbance.

3) A network of inertial traverses throughout a region with numerous crossover points obtains high relative accuracy in an adjustment with simple parametric models for systematic error.

4) Control should be introduced into network adjustments with proper covariances, due to the high relative accuracy of the inertial measurements.

5) The relative accuracies obtainable in nominal network surveys with inertial systems currently appear to be from 10 to 30 cm in level position and elevation and from .3 to 4 arc-s for the deflection of the vertical. In fact, accuracies relative to existing control at the upper end of this range have tended to cast doubts on the accuracy of the control.

Future Investigations

Given the above conclusions, it appears that investigations into the following issues would be useful to obtain performance enhancement for the user of inertial survey equipment. For the user whose work does not allow the use of a network approach but collects measurements with forward and reverse traverses:

1) Determine any performance enhancement obtainable by the use of "suspended" states for errors in the estimates of position and the gravity disturbance at survey points on the forward traverse which can then be processed with the comparison measurements at the same points on the reverse traverse. As a Kalman filter-smoother problem, such a study would provide a near-optimal reference for deterministic empirical smoothers, which today are processing these measurements to reduce the effects of error instabilities due to various sources. Recall, however, that the effect of the spatially correlated gravity disturbance is not truly represented as time-correlated stochastic process in a simple manner here.

2) Determine the performance enhancement of an ad hoc technique in which the estimates of the gravity disturbance vector obtained on the forward traverse, and enhanced by traverse closure measurements, can be used to reduce the error effect of the gravity disturbance vector on the reverse traverse.

For the user whose work does allow the use of network techniques, answers to the above questions would also be useful in that they could improve the quality of the data to be used in the network adjustment. Further, however, the error effect of the underlying regionally correlated gravity-disturbance vector has not yet been clearly quantified. Specifically, the effect on the final adjustment results of the correlation of error between survey measurements obtained on spatially near traverses, due to the gravity disturbance, needs to be addressed.

Conclusion

It is clear from a review of the literature on the field of inertial survey that this is an enterprise of significant commercial and military importance where optimal estimation techniques have made a major contribution to success. The experimental and theoretical work performed over the last five years has indicated that some of the original estimation approaches are near optimal in performance, while remaining attractive in their implementation cost. For a single traverse, theoretical relative position accuracies of a few parts per million are being approached by some users. For regional surveys comprised of a number of intersecting traverses, however, some theoretical issues remain as indicated above, which when resolved should lead to further improvements in performance. Further, as we look to the future it is clear that more accurate inertial equipment will be available for survey use. This equipment, in conjunction with developments in other areas of gravity gradiometry and satellite range and doppler measurement, will in some as-yet-to-be-determined combination lead to substantially improved accuracy, while retaining the high production rates available today with inertial survey systems.

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